Appendix
to “Measures of rule interestingness in various perspectives of confirmation”

Contents

For the sake of presentation simplicity and to make the Appendix self-contained, the enumeration of Tables, scenarios and equations in the Appendix is independent of the enumeration in the main text.

A1. Proof of Theorem 1

Theorem 1. A confirmation measure being strictly monotonic in a given perspective, $c(H,E) = f(\Pr_L, \Pr_R)$ with f being strictly increasing in the first argument and strictly decreasing in the second argument, is not strictly monotonic in the other perspectives.

Proof. Consider scenario 1 corresponding to the contingency table in Table 1 and scenario 2 corresponding to the contingency table in Table 2.

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Table 1. Contingency table for scenario 1

	H_1	$\neg H_1$	$\sum$
E_1	a_1=10	c_1=2	a_1+c_1=12
$\neg E_1$	b_1=10	d_1=78	b_1+d_1=88
$\sum$	a_1+b_1=20	c_1+d_1=80	$\|U\|$=100

Table 2. Contingency table for scenario 2

	H_2	$\neg H_2$	$\sum$
E_2	a_2=20	c_2=4	a_2+c_2=24
$\neg E_2$	b_2=0	d_2=76	b_2+d_2=76
$\sum$	a_2+b_2=20	c_2+d_2=80	$\|U\|$=100

Observe that

$$\Pr(H_1 \mid E_1) = \frac{a_1}{a_1 + c_1} = \frac{a_2}{a_2 + c_2} = \Pr(H_2 \mid E_2) = 0.833$$

and $$\Pr(H_1) = \frac{a_1 + b_1}{a_1 + b_1 + c_1 + d_1} = \frac{a_2 + b_2}{a_2 + b_2 + c_2 + d_2} = \Pr(H_2) = 0.2$$

and therefore for any confirmation measure $c_{smB}(H,E) = f_{smB}(\Pr(H|E), \Pr(H))$ strictly monotonic in the Bayesian perspective we get:

$$c_{smB}(H_1,E_1) = c_{smB}(H_2,E_2) \tag{1}$$

Observe also that

$$\Pr(H_1 \mid \neg E_1) = \frac{b_1}{b_1 + d_1} = 0.114 \text{ and } \Pr(H_2 \mid \neg E_2) = \frac{b_2}{b_2 + d_2} = 0 .$$

Thus, having

$$\frac{a_1}{a_1+c_1}=\frac{a_2}{a_2+c_2} \text{ and } \frac{b_1}{b_1+d_1}>\frac{b_2}{b_2+d_2},$$

for any confirmation measure $c_{smSB}(H,E) = f_{smSB}(\Pr(H|E), \Pr(H|\neg E))$ strictly monotonic in the strong Bayesian perspective we get:

$$c_{smSB}(H_1,E_1) < c_{smSB}(H_2,E_2) \tag{2}$$

As a result, $c_{smSB}(H,E)$ cannot respect the strict monotonicity in the Bayesian perspective as by (1) it should give $c_{smSB}(H_1,E_1) = c_{smSB}(H_2,E_2)$ instead of (2).

With analogous examples one can prove the thesis for all the remaining couples of perspectives of confirmation.

Let us conclude the proof, by showing that the hypothesis that f in $c(H,E) = f(\Pr_L, \Pr_R)$ is strictly increasing in the first argument and strictly decreasing in the second argument is necessary and cannot be removed. Let us observe that there exists a confirmation measure $c_U(H,E)$ which is "universally" monotonic in all the four perspectives:

$$c_U(H,E)=\begin{cases}1 & \text{if } ad-bc>0,\\ 0 & \text{if } ad-bc=0,\\ -1 & \text{if } ad-bc<0.\end{cases} \tag{3}$$

It is easy to see that one can write $c_U(H,E)=f(\Pr_L, \Pr_R)$ in terms of left- and right-hand side probabilities $\Pr_L$ and $\Pr_R$ of any of the four perspectives as follows:

$$c_U(H,E)=\begin{cases}1 & \text{if } \Pr_L>\Pr_R,\\ 0 & \text{if } \Pr_L=\Pr_R,\\ -1 & \text{if } \Pr_L<\Pr_R.\end{cases} \tag{4}$$

The confirmation measure $c_U(H,E)$ succeeds in respecting the monotonicity in all the four perspectives because in any perspective it is non-decreasing in the first argument and non-increasing in the second argument, rather than being strictly increasing in the first argument and strictly decreasing in the second argument, as required by the hypothesis of Theorem 1. □

A2. Proof of Observation 1

Observation 1. Measure $V(H,E)$ respects monotonicity M.

Proof. Measure $V(H,E)$ is defined in the following way:

$$V(H,E)=\begin{cases}\dfrac{\Pr(E\mid H)-\Pr(E)}{1-\Pr(E)}=\dfrac{ad-bc}{(a+b)(b+d)} & \text{in case of confirmation}\\ \dfrac{\Pr(E\mid H)-\Pr(E)}{\Pr(E)}=\dfrac{ad-bc}{(a+c)(a+b)} & \text{in case of disconfirmation}\end{cases}$$

To show that it respects monotonicity M, let us first consider $V(H,E)$ in case of confirmation, i.e., when $ad\text{-}bc > 0$. Let us verify if $V(H,E)$ is non-decreasing with respect to a, i.e., if an increase of a by $\Delta>0$ will not result in decrease of $V(H,E)$. Simple algebraic transformations show that:

$$\frac{(a+\Delta)d-bc}{(a+\Delta+b)(b+d)}-\frac{ad-bc}{(a+b)(b+d)}=\frac{bd\Delta+bc\Delta}{(a+\Delta+b)(b+d)(a+b)}\geq 0\,.$$

Thus, $V(H,E)$ (in case of confirmation) is non-decreasing with respect to a.

Now, let us verify if $V(H,E)$ is non-increasing with respect to b, i.e., if an increase of b by $\Delta>0$ will not result in increase of $V(H,E)$. Simple algebraic transformations show that:

$$\frac{ad-(b+\Delta)c}{(a+b+\Delta)(b+\Delta+d)}-\frac{ad-bc}{(a+b)(b+d)}$$
$$=\frac{(b\Delta+\Delta^2)(bc-ad)-ad(a\Delta+b\Delta+c\Delta+d\Delta)}{(a+b+\Delta)(b+\Delta+d)(a+b)(b+d)}\leq 0$$

This is because: $b\Delta+\Delta^2\geq 0$, $ad(a\Delta+b\Delta+c\Delta+d\Delta)\geq 0$, and $(bc\text{-}ad)<0$ as we consider the case of confirmation. Thus, we can conclude that $V(H,E)$ (in case of confirmation) is non-increasing with respect to b.

Clearly, $V(H,E)$ is also non-increasing with respect to c, as increase of c by $\Delta>0$ will result in decrease of the numerator: $ad\text{-}bc$ (while the denominator: $(a+b)(b+d)$ remains unchanged) and therefore in decrease of $V(H,E)$.

Finally, let us verify if $V(H,E)$ is non-decreasing with respect to d, i.e., if an increase of d by $\Delta>0$ will not result in decrease of $V(H,E)$. Simple algebraic transformations show that:

$$\frac{a(d+\Delta)-bc}{(a+b)(b+d+\Delta)}-\frac{ad-bc}{(a+b)(b+d)}=\frac{ab\Delta+bc\Delta}{(a+b)(b+d)(b+d+\Delta)}\geq 0$$

Thus, $V(H,E)$ (in case of confirmation) is non-decreasing with respect to d.

Since all four conditions are satisfied, the hypothesis that measure $V(H,E)$ in case of confirmation has the property of monotonicity M is true. The proof that measure $V(H,E)$ satisfies the property of monotonicity M in case of disconfirmation is the same as for measure $Z(H,E)$ in case of disconfirmation (see Greco, S., Słowiński, R., Szczęch, I., 2008. Assessing the quality of rules with a new monotonic interestingness measure Z. In: Rutkowski, L., Tadeusiewicz, R., Zadeh, L.A., Zurada, J.M. (eds.), *Artificial Intelligence and Soft Computing* (*ICAISC 2008)*, LNAI, vol. 5097, pp. 556-565. Springer, *H*eidelberg).

A3. Proof of Theorem 2

Theorem 2. Under weak Ex_1, confirmation measure $c(H,E)$ cannot attain its maximum value unless $E \models H$ or $\neg E \models \neg H$, i.e., c=0 or b=0. Confirmation measure $c(H,E)$ satisfying weak Ex_1 cannot attain its minimum value unless $E \models \neg H$ or $\neg E \models H$, i.e., a=0 or d=0.

Proof. Suppose, by contradiction, that $c(H,E)$ attains its maximum value but it is not true that $E \models H$ nor $\neg E \models \neg H$. This means that not $(E \models H)$ and not $(\neg E \models \neg H)$, which means that c>0 and b>0.

Consider now E^* and H^* such that $E^* \models H^*$ and $\neg E^* \models \neg H^*$. By weak Ex_1, we get: $c(H^*,E^*) > c(H,E)$, which makes a contradiction, because $c(H,E)$ was supposed to attain its maximum value.

By definition, $E \models H$ is equivalent to c=0, and $\neg E \models \neg H$ is equivalent to b=0. Thus, we conclude that $c(H,E)$ cannot attain its maximum value unless $E \models H$ or $\neg E \models \neg H$. Analogous proof holds for the case in which $c(H,E)$ attains its minimum value. □

A4. Proof of Theorem 3

Theorem 3. Under weak L-Ex_1, confirmation measure $c(H,E)$ cannot attain its maximum value unless $H \models E$ or $\neg H \models \neg E$, i.e., b=0 or c=0. Confirmation measure $c(H,E)$ satisfying weak L-Ex_1 cannot attain its minimum value unless $H \models \neg E$ or $\neg H \models E$, i.e., a=0 or d=0.

Proof. Let us observe that Theorem 3 is the counterpart of Theorem 2 with respect to weak L-Ex_1 property. Thus, we give it without proof because it can be proved analogously to Theorem 2.

A5. Proof of Theorem 4

Theorem 4. Confirmation measures strictly monotonic in the strong Bayesian perspective satisfy weak Ex_1 property. Confirmation measures strictly monotonic in the strong likelihoodist perspective satisfy weak L-Ex_1 property.

Proof. The condition $v(H_A,E_A)>v(H_B,E_B)$ can be satisfied in the following cases

a) $v(H_A,E_A)=V$ and $v(H_B,E_B)\leq 0$, which means $E_A|=H_A$ and not $E_B|=H_B$, which implies $\Pr(H_A|E_A)=1$ and $\Pr H_B|E_B)<1$;
b) $v(H_A,E_A)=0$ and $v(H_B,E_B)= -V$, which means neither $E_A|= H_A$ nor $E_A|=\neg H_A$, and $E_B|=\neg H_B$, which implies $0< \Pr(H_A|E_A)<1$ and $\Pr(H_B|E_B)=0$.

Analogously, the condition $v(H_A, \neg E_A)<v(H_B, \neg E_B)$ can be satisfied in the following cases

c) $v(H_A,\neg E_A)\leq 0$ and $v(H_B, \neg E_B)=V$, which means not $\neg E_A|= H_A$ and $\neg E_B|=H_B$, which implies $\Pr(H_A| \neg E_A)<1$ and $\Pr(H_B|\neg E_B)=1$;
d) $v(H_A,\neg E_A)= -V$ and $v(H_B|\neg E_B)=0$, which means $\neg E_A|=\neg H_A$, and neither $\neg E_B|= H_B$ nor $\neg E_B|= \neg H_B$, which implies $\Pr(H_A| \neg E_A)=0$ and $0< \Pr(H_B|\neg E_B)<1$.

Therefore, the premise of weak Ex_1, that is $v(H_A,E_A)>v(H_B,E_B)$ and $v(H_A, \neg E_A)<v(H_B, \neg E_B)$, is satisfied in the four following cases:

I. a) and c),
II. a) and d),
III. b) and c),
IV. b) and d).

In all the cases I-IV we have $\Pr(H_A|E_A) > \Pr(H_B|E_B)$ and $\Pr(H_A|\neg E_A) < \Pr(H_B|\neg E_B)$. Thus, by strict monotonicity in the strong Bayesian confirmation we get $c(H_A, E_A) > c(H_B, E_B)$. Thus, we proved that confirmation measures being strictly monotonic in the strong Bayesian perspective satisfy the weak Ex_1 property. With analogous proof it can be shown that confirmation measures strictly monotonic in the strong likelihoodist perspective satisfy weak L-Ex_1 property. □

A6. Proof of Theorem 5

Theorem 5. Confirmation measures monotonic in the strong Bayesian perspective and measures monotonic in the strong likelihoodist perspective satisfy weak L property. Confirmation measures strictly monotonic in the strong Bayesian perspective, as well as confirmation measures strictly monotonic in the strong likelihoodist perspective, satisfy maximality/minimality.

Proof. Suppose that $c_{SB}(H, E)$ is a confirmation measure monotonic in the strong Bayesian perspective. Suppose also that for hypothesis H_α and evidence E_α we have $E_\alpha \models H_\alpha$ and $\neg E_\alpha \models \neg H_\alpha$. Observe that $E_\alpha \models H_\alpha$ implies that $\Pr(H_\alpha|E_\alpha)=1$ as well as $\neg E_\alpha \models \neg H_\alpha$ implies that $\Pr(H_\alpha|\neg E_\alpha)=0$. This means that $\Pr(H_\alpha|E_\alpha)$ attains the maximum value for the left-hand side probability of the strong Bayesian perspective as well $\Pr(H_\alpha|\neg E_\alpha)$ attains the minimum value for the right-hand side probability of the strong Bayesian perspective. Thus, for the monotonicity in the strong Bayesian perspective, $c_{SB}(H_\alpha, E_\alpha)$ must be maximal.

With an analogous proof, supposing that for hypothesis H_β and evidence E_β we have $E_\beta \models \neg H_\beta$ and $\neg E_\beta \models H_\beta$, we can show that $c_{SB}(H_\beta, E_\beta)$ must be minimal. Thus, we proved that confirmation measures monotonic in the strong Bayesian perspective satisfy weak L property.

Suppose now that $c_{SL}(H, E)$ is a confirmation measure monotonic in the strong likelihoodist perspective. Taking again the above hypothesis H_α and evidence E_α, observe that $E_\alpha \models H_\alpha$ implies that $\neg H_\alpha \models \neg E_\alpha$ and, consequently, $\Pr(E_\alpha|\neg H_\alpha)=0$, as well as $\neg E_\alpha \models \neg H_\alpha$ implies that $H_\alpha \models E_\alpha$ and, consequently, $\Pr(E_\alpha|H_\alpha)=1$ This means that $\Pr(E_\alpha|H_\alpha)$ attains the maximum value for the left-hand side probability of the strong likelihoodist perspective as well $\Pr(E_\alpha|\neg H_\alpha)$ attains the minimum value for the right-hand side probability of the strong likelihoodist perspective. Thus, for the monotonicity in the strong likelihoodist perspective, $c_{SL}(H_\alpha, E_\alpha)$ must be maximal.

With an analogous proof, supposing that for hypothesis H_β and evidence E_β we have $E_\beta \models \neg H_\beta$ and $\neg E_\beta \models H_\beta$, we can show that $c_{SL}(H_\beta, E_\beta)$ must be minimal. Thus, we proved that confirmation measures monotonic in the strong likelihoodist perspective satisfy weak L property.

Observe that $E \models H$ is equivalent to $c = 0$ and $\neg E \models \neg H$ is equivalent to $b = 0$. Thus the first part of the theorem proves that a confirmation measure monotonic in the strong Bayesian perspective being strictly increasing with respect to $\Pr(H|E)$ and strictly decreasing with respect to $\Pr(H|\neg E)$ attain their maximal value if $b = c = 0$ (in fact, knowing that the confirmation measure is monotonic with respect to left- and right-hand side probabilities of the strong Bayesian perspective is sufficient for this).

Thus, we have to prove that for confirmation measures $c_{smSB}(H,E)$ being strictly monotonic in the strong Bayesian perspective, i.e. strictly increasing with respect to $\Pr(H|E)$ and strictly decreasing with respect to $\Pr(H|\neg E)$, if they attain their maximal value then $b = c = 0$. By contradiction suppose that for some hypothesis H_γ and some evidence E_γ, we have $b_\gamma>0$ and $c_\gamma= 0$, and nevertheless $c_{smSB}(H_\gamma,E_\gamma)$ does attain its maximal value. Then we can consider some hypothesis H_δ and some evidence E_δ such that $b_\delta=0$ and $c_\delta= 0$. This implies that $\Pr(H_\gamma|E_\gamma)=\Pr(H_\delta|E_\delta)=1$ and $\Pr(H_\gamma|\neg E_\gamma)>\Pr(H_\delta|\neg E_\delta)=0$, so that, by the strict monotonicity with respect to left- and right-hand side probabilities of the strong Bayesian perspective we get $c_{smSB}(H_\gamma,E_\gamma)< c_{smSB}(H_\delta,E_\delta)$. But this means that $c_{smSB}(H_\gamma,E_\gamma)$ cannot be maximal, which is a contradiction. Thus we proved that confirmation measures $c_{smSB}(H,E)$ cannot attain its maximal value if $b>0$.

Analogously, we can prove that confirmation measures $c_{smSB}(H,E)$ cannot attain its maximal value if $c>0$, and that $c_{smSB}(H,E)$ attain its minimal value if and only if $a>0$ or $d>0$. Thus we proved that confirmation measures $c_{smSB}(H,E)$ strictly monotonic in the strong Bayesian perspective satisfy maximality/minimality.

Similarly we can prove that confirmation measures $c_{smSL}(H,E)$ strictly monotonic in the strong likelihoodist perspective, i.e. strictly increasing with respect to $\Pr(E|H)$ and strictly decreasing with respect to $\Pr(E|\neg H)$, satisfy maximality/minimality. □

A7. Proof of Theorem 6

Theorem 6. Consider a confirmation measure $c_{smB}(H, E)$ strictly monotonic in the Bayesian perspective. The following statements hold:

1) there are no confirmation measures $c_{smB}(H, E)$ that satisfy *ES*;
2) there exist confirmation measures $c_{smB}(H, E)$ that satisfies *HS*;
3) there are no confirmation measures $c_{smB}(H, E)$ that satisfy *EIS*;
4) there are no confirmation measures $c_{smB}(H, E)$ that satisfy *HIS*;
5) there exist confirmation measures $c_{smB}(H, E)$ that satisfies *IS*;
6) there are no confirmation measures $c_{smB}(H, E)$ that satisfy *EHS*;
7) there exist confirmation measures $c_{smB}(H, E)$ that satisfies *EHIS*.

Moreover, if $c_{smB}(H, E)$ satisfies one among *HS*, *IS* and *EHIS*, it cannot satisfy any of the remaining two symmetry properties. Finally, there are confirmation measures $c_{smB}(H, E)$ that do not satisfy any symmetry property.

Proof. Before proving one by one, all the points of Theorem 6 observe that by hypothesis $c_{smB}(H, E)$ is a confirmation measure strictly monotonic in the Bayesian perspective. Thus, there exists a function f_{smB}:[0,1]×[0,1]→$\mathfrak{R}$ strictly increasing with respect to the first argument and strictly decreasing with respect to the second argument, with $f_{smB}(x,x) = 0$ for all $x\in[0,1]$, such that

$$c_{smB}(H,E) = f_{smB}(\Pr(H|E), \Pr(H)).$$

1) **Evidence symmetry**

Observe that if a confirmation measure $c_{smB}(H, E)$ satisfies *ES*, then for any hypothesis H and any evidence E we have:

$$c_{smB}(H, E) = -c_{smB}(H, \neg E).$$

Now, consider scenarios 3 and 4 represented in Table 3 and Table 4, respectively.

Table 3. Contingency tables for scenario 3

	H_3	$\neg H_3$	$\sum$
E_3	a_3=10	c_3=30	a_3+c_3=40
$\neg E_3$	b_3=20	d_3=40	b_3+d_3=60
$\sum$	a_3+b_3=30	c_3+d_3=70	$\|U\|$=100

Table 4. Contingency tables for scenario 4

	H_4	$\neg H_4$	$\sum$
E_4	a_4=3	c_4=9	a_4+c_4=12
$\neg E_4$	b_4=3	d_4=5	b_4+d_4=8
$\sum$	a_4+b_4=6	c_4+d_4=14	$\|U\|$=20

For scenario 3 we have $\Pr(H_3|E_3)$=0.25, $\Pr(H_3)$=0.3, $\Pr(H_3|\neg E_3)=\frac{1}{3}$, and for scenario 4 we have $\Pr(H_4|E_4)$=0.25, $\Pr(H_4)$=0.3, $\Pr(H_4|\neg E_4)=\frac{3}{8}$.

Taking into account that $\Pr(H_3|E_3)=\Pr(H_4|E_4)$ and $\Pr(H_3)=\Pr(H_4)$, we get

$$\begin{aligned} c_{smB}(H_3,E_3) = f_{smB}(\Pr(H_3 \mid E_3),\Pr(H_3)) = \\ f_{smB}(\Pr(H_4 \mid E_4),\Pr(H_4)) = c_{smB}(H_4,E_4). \end{aligned} \quad (5)$$

For contradiction, suppose that $c_{smB}(H,E)$ satisfies *ES*. This would imply that

$$c_{smB}(H_3,E_3) = -c_{smB}(H_3,\neg E_3) \tag{6}$$

$$c_{smB}(H_4,E_4) = -c_{smB}(H_4,\neg E_4) \tag{7}$$

Thus, by (5), (6) and (7) it should be: $c_{smB}(H_3, \neg E_3)= c_{smB}(H_4, \neg E_4)$,
that is:

$$f_{smB}(\Pr(H_3 \mid \neg E_3),\Pr(H_3)) = f_{smB}(\Pr(H_4 \mid \neg E_4),\Pr(H_4)). \tag{8}$$

Observing that $f_{smB}(\Pr(H|E), \Pr(H))$ is strictly increasing in the first argument and since $\Pr(H_3|\neg E_3) < \Pr(H_4|\neg E_4)$ while $\Pr(H_3)= \Pr(H_4)$, we obtain

$$f_{smB}(\Pr(H_3 \mid \neg E_3),\Pr(H_3)) < f_{smB}(\Pr(H_4 \mid \neg E_4),\Pr(H_4)). \tag{9}$$

But (8) and (9) are contradictory and therefore $c_{smB}(H,E)$ cannot satisfy the evidence symmetry *ES*.

2) **Hypothesis symmetry**

Observe that if a confirmation measure $c_{smB}(H, E)$ satisfies *HS*, then for any hypothesis *H* and any evidence *E* we have:

$$c_{smB}(H, E) =-c_{smB}(\neg H, E)$$

A family of confirmation measures strictly monotonic in the Bayesian perspective $c_{smB}(H, E)$ satisfying *HS* is the following:

$$c_{smB}(H, E) = g\big(\Pr(H \mid E) - \Pr(H)\big)$$

with $g:\mathfrak{R} \to \mathfrak{R}$ being a strictly increasing odd function (i.e., $g(-x)=-g(x)$). Indeed for any hypothesis *H* and any evidence *E* we have:

$\Pr(H|E) - \Pr(H)= - [(1- \Pr(H|E)) - (1- \Pr(H))] =-[\Pr(\neg H|E)-\Pr(\neg H)]$

so that, for $g(x)$ being an odd function, we get

$c_{smB}(H, E)= g\big(\Pr(H \mid E) - \Pr(H)\big)=- g\big(\Pr(\neg H \mid E) - \Pr(\neg H)\big)=-c_{smB}(\neg H, E)$.

3) **Evidence-inversion symmetry**

Observe that if a confirmation measure $c_{smB}(H, E)$ satisfies *EIS*, then for any hypothesis *H* and any evidence *E*, by applying the *EIS* twice, we have:

$$c_{smB}(H,E) = -c_{smB}(\neg E,H) = c_{smB}(\neg H,\neg E) \tag{10}$$

Thus, taking the first and the last terms of (10) and considering function $f_{smB}(\Pr(H|E), \Pr(H))$, we get

$$f_{smB}(\Pr(H \mid E), \Pr(H)) = f_{smB}(\Pr(\neg H \mid \neg E), \Pr(\neg H)). \tag{11}$$

Taking into account the above scenarios 3 and 4 in Table 3 and Table 4, respectively, we have $\Pr(H_3|E_3)=0.25$, $\Pr(H_3)=0.3$, $\Pr(\neg H_3|\neg E_3)=\frac{2}{3}$, $\Pr(\neg H_3)=0.7$ and for scenario 4 we have $\Pr(H_4|E_4)=0.25$, $\Pr(H_4)=0.3$, $\Pr(\neg H_4|\neg E_4)=\frac{5}{8}$, $\Pr(\neg H_4)=0.7$.

As observed in the proof of above point 1), we have

$$\begin{aligned} &c_{smB}(H_3, E_3) = f_{smB}(\Pr(H_3 \mid E_3), \Pr(H_3)) = \\ &f_{smB}(\Pr(H_4 \mid E_4), \Pr(H_4)) = c_{smB}(H_4, E_4). \end{aligned} \tag{12}$$

If for contradiction we suppose that $c_{smB}(H,E)$ satisfies *EIS*, by (10) we should have

$$c_{smB}(H_3, E_3) = c_{smB}(\neg H_3, \neg E_3) \tag{13}$$

$$c_{smB}(H_4, E_4) = c_{smB}(\neg H_4, \neg E_4) \tag{14}$$

Thus, by (12), (13) and (14) it should be: $c_{smB}(\neg H_3, \neg E_3)= c_{smB}(\neg H_4, \neg E_4)$, that is:

$$f_{smB}(\Pr(\neg H_3 \mid \neg E_3), \Pr(\neg H_3)) = f_{smB}(\Pr(\neg H_4 \mid \neg E_4), \Pr(\neg H_4)). \tag{15}$$

Observing that $f_{smB}(\Pr(H|E), \Pr(H))$ is strictly increasing in the first argument and since $\Pr(\neg H_3|\neg E_3) > \Pr(\neg H_4|\neg E_4)$ while $\Pr(\neg H_3)= \Pr(\neg H_4)$, we would obtain

$$f_{smB}(\Pr(\neg H_3 \mid \neg E_3), \Pr(\neg H_3)) > f_{smB}(\Pr(\neg H_4 \mid \neg E_4), \Pr(\neg H_4)). \tag{16}$$

But (15) and (16) are contradictory which implies that $c_{smB}(H,E)$ cannot satisfy the evidence-inversion symmetry *EIS*.

4) **Hypothesis-inversion symmetry**
Observe that if a confirmation measure $c_{smB}(H, E)$ satisfies *HIS*, then for any hypothesis *H* and any evidence *E,* by applying the *HIS* twice, we have:

$$c_{smB}(H,E) = -c_{smB}(E,\neg H) = c_{smB}(\neg H,\neg E) \tag{17}$$

Since by (17) $c_{smB}(H, E) = c_{smB}(\neg H, \neg E)$ as in the above proof for point 3) (see equation (10)), by the same argument used to prove that $c_{smB}(H, E)$ cannot satisfy *EIS*, we get that $c_{smB}(H, E)$ cannot satisfy the hypothesis-inversion symmetry *HIS*.

5)**Inversion symmetry**
Observe that if a confirmation measure $c_{smB}(H, E)$ satisfies *IS*, then for any hypothesis *H* and any evidence *E* we have:

$$c_{smB}(H, E) = c_{smB}(E,H)$$

A family of confirmation measures $c_{msB}(H, E)$ strictly monotonic in the Bayesian perspective, satisfying *IS* is the following:

$$c_{smB}(H,E) = g\left(\frac{\Pr(H \mid E) - \Pr(H)}{\Pr(H \mid E) + \Pr(H)}\right)$$

with g: $\mathfrak{R} \rightarrow \mathfrak{R}$ being a strictly increasing odd function.
Indeed for any hypothesis *H* and any evidence *E* we have:

$$c_{smB}(H,E) = g\left(\frac{\Pr(H \mid E) - \Pr(H)}{\Pr(H \mid E) + \Pr(H)}\right) = g\left(\frac{\dfrac{\Pr(H \mid E)}{\Pr(H)} - 1}{\dfrac{\Pr(H \mid E)}{\Pr(H)} + 1}\right)$$

$$= g\left(\frac{\dfrac{\Pr(E \mid H)}{\Pr(E)} - 1}{\dfrac{\Pr(E \mid H)}{\Pr(E)} + 1}\right) = g\left(\frac{\Pr(E \mid H) - \Pr(E)}{\Pr(E \mid H) + \Pr(E)}\right) = c_{smB}(E,H).$$

6) **Evidence-hypothesis symmetry**
The same argument used in the above proof of point 3) and 4) showing that $c_{smB}(H, E)$ cannot satisfy *EIS* and *HIS* proves that $c_{smB}(H, E)$ cannot satisfy the evidence-hypothesis symmetry *EHS*.

7) **Evidence-hypothesis-inversion symmetry**

Observe that if a confirmation measure $c_{smB}(H, E)$ satisfies *EHIS*, then for any hypothesis H and any evidence E we have:

$$c_{smB}(H, E) = c_{smB}(\neg E, \neg H)$$

A family of confirmation measures $c_{smB}(H, E)$ strictly monotonic in the Bayesian perspective, satisfying *EHIS* is the following:

$$c_{smB}(H,E) = g\left(\frac{\Pr(H \mid E) - \Pr(H)}{2 - \Pr(H \mid E) - \Pr(H)}\right)$$

with g:$\mathfrak{R} \rightarrow \mathfrak{R}$ being a strictly increasing odd function.

Indeed for any hypothesis H and any evidence E we have:

$$c_{smB}(H,E) = g\left(\frac{\Pr(H \mid E) - \Pr(H)}{2 - \Pr(H \mid E) - \Pr(H)}\right)$$

$$= g\left(\frac{(1 - \Pr(H)) - (1 - \Pr(H \mid E))}{(1 - \Pr(H)) + (1 - \Pr(H \mid E))}\right)$$

$$= g\left(\frac{\Pr(\neg H) - \Pr(\neg H \mid E)}{\Pr(\neg H) + \Pr(\neg H \mid E)}\right) = g\left(\frac{1 - \dfrac{\Pr(\neg H \mid E)}{\Pr(\neg H)}}{1 + \dfrac{\Pr(\neg H \mid E)}{\Pr(\neg H)}}\right)$$

$$= g\left(\frac{1 - \dfrac{\Pr(E \mid \neg H)}{\Pr(E)}}{1 + \dfrac{\Pr(E \mid \neg H)}{\Pr(E)}}\right) = g\left(\frac{\Pr(E) - \Pr(E \mid \neg H)}{\Pr(E) + \Pr(E \mid \neg H)}\right)$$

$$= g\left(\frac{(1 - \Pr(E \mid \neg H)) - (1 - \Pr(E))}{2 - (1 - \Pr(E \mid \neg H)) + (1 - \Pr(E))}\right)$$

$$= g\left(\frac{\Pr(\neg E \mid \neg H) - \Pr(\neg E)}{2 - \Pr(\neg E \mid \neg H) - \Pr(\neg E)}\right) = c_{smB}(\neg E, \neg H).$$

Let now prove that if $c_{smB}(H, E)$ satisfies one of: *HS*, *IS* or *EHIS*, then it cannot satisfy any of the other two remaining symmetry properties. Let us consider one by one the three possible couples of properties from *HS*, *IS* and *EHIS*.

- By contradiction suppose that $c_{smB}(H, E)$ satisfies properties *HS* and *IS*. Applying first *HS* and after *IS*, we get

  $$c_{smB}(H, E) = -c_{smB}(\neg H, E) = -c_{smB}(E, \neg H).$$

  Thus, taking into account the first and the last term, we have that $c_{smB}(H, E)$ satisfies *HIS*. But by the above point 4) this is impossible and, therefore, it is also not possible that $c_{smB}(H, E)$ satisfies at the same time both *HS* and *IS* properties.

- By contradiction suppose that $c_{smB}(H, E)$ satisfies properties *HS* and *EHIS*. Applying first *HS* and after *EHIS*, we get
$$c_{smB}(H, E) = -c_B(\neg H, E) = -c_B(\neg E, H).$$
Thus, taking into account the first and the last term, we have that $c_{smB}(H, E)$ satisfies *EIS*. But by the above point 3) this is impossible and, therefore, it is also not possible that $c_{smB}(H, E)$ satisfies at the same time both *HS* and *EHIS* properties.
- By contradiction suppose that $c_{smB}(H, E)$ satisfies properties *IS* and *EHIS*. Applying first *IS* and after *EHIS*, we get
$$c_{smB}(H, E) = c_B(E, H) = c_B(\neg H, \neg E).$$
Thus, taking into account the first and the last term, we have that $c_{smB}(H, E)$ satisfies *EHS*. But by above point 6) this is impossible and, therefore, it is also not possible that $c_{smB}(H, E)$ satisfies at the same time both *IS* and *EHIS* properties.

Finally, let us now show that there are confirmation measures monotonic in the Bayesian perspective that do not satisfy any symmetry property. Consider the confirmation measure
$$D^*(H, E) = \sqrt{\Pr(H \mid E)} - \sqrt{\Pr(H)}.$$
Indeed

- $D^*(H, E)$ does not satisfy *HS* because, in general
$$D^*(H, E) = \sqrt{\Pr(H \mid E)} - \sqrt{\Pr(H)} \neq -\left(\sqrt{1-\Pr(H \mid E)} - \sqrt{1-\Pr(H)}\right) = -\left(\sqrt{\Pr(\neg H \mid E)} - \sqrt{\Pr(\neg H)}\right) = -\ D^*(\neg H, E)\,,$$
- $D^*(H, E)$ does not satisfy *IS* because, in general
$$D^*(H, E) = \sqrt{\Pr(H \mid E)} - \sqrt{\Pr(H)} \neq \sqrt{\Pr(E \mid H)} - \sqrt{\Pr(E)} = D^*(E,H)\,,$$
- $D^*(H, E)$ does not satisfy *EHIS* because, in general
$$D^*(H, E) = \sqrt{\Pr(H \mid E)} - \sqrt{\Pr(H)} \neq \sqrt{\Pr(\neg E \mid \neg H)} - \sqrt{\Pr(\neg E)} = D^*(\neg E, \neg H)\,,$$
- the other symmetry properties cannot be satisfied, because we have already proved that there is no confirmation measure monotonic in the Bayesian perspective satisfying those symmetry properties. □

A8. Proof of Theorem 7

Theorem 7. Consider a confirmation measure $c_{smSB}(H, E)$ strictly monotonic in the strong Bayesian perspective. For any symmetry property there are confirmation measures $c_{smSB}(H, E)$ satisfying it. Moreover, there are confirmation measures $c_{smSB}(H, E)$ that satisfy all symmetry properties.

Proof.

1) **Evidence symmetry**

Observe that if a confirmation measure $c_{smSB}(H, E)$ satisfies *ES*, then for any hypothesis *H* and any evidence *E* we have:

$$c_{smSB}(H, E) = -c_{smSB}(H, \neg E).$$

A family of confirmation measures $c_{smSB}(H, E)$ that satisfy *ES* is the following:

$$c_{smSB}(H,E) = g\big(\Pr(H \mid E) - \Pr(H \mid \neg E)\big)$$

with g:$\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function.

Indeed for any hypothesis *H* and any evidence *E* we have:

$$\begin{aligned} c_{smSB}(H,E) &= g\big(\Pr(H \mid E) - \Pr(H \mid \neg E)\big) \\ &= -g\big(\Pr(H \mid \neg E) - \Pr(H \mid E)\big) = -c_{smSB}(H, \neg E). \end{aligned}$$

2) **Hypothesis symmetry**

Observe that if a confirmation measure $c_{smSB}(H, E)$ satisfies *HS*, then for any hypothesis *H* and any evidence *E* we have:

$$c_{smSB}(H, E) = -c_{smSB}(\neg H, E).$$

A family of confirmation measures $c_{smSB}(H, E)$ that satisfy *HS* is the following:

$$c_{smSB}(H,E) = g\big(\Pr(H \mid E) - \Pr(H \mid \neg E)\big)$$

with g:$\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function.

Indeed for any hypothesis *H* and any evidence *E* we have:

$$\begin{aligned} c_{smSB}(H,E) &= g\big(\Pr(H \mid E) - \Pr(H \mid \neg E)\big) \\ &= -g\big(\big(1 - \Pr(H \mid E)\big) - \big(1 - \Pr(H \mid \neg E)\big)\big) \\ &= -g\big(\Pr(\neg H \mid E) - \Pr(\neg H \mid \neg E)\big) = -c_{smSB}(\neg H, E). \end{aligned}$$

3) **Evidence-inversion symmetry**

Observe that if a confirmation measure $c_{smSB}(H, E)$ satisfies *EIS*, then for any hypothesis *H* and any evidence *E* we have:

$$c_{smSB}(H, E) = -c_{smSB}(\neg E, H).$$

A family of confirmation measures $c_{smSB}(H, E)$ that satisfy *EIS* is the following:

$$c_{smSB}(H,E)=g\left(\frac{\Pr(H\mid E)(1-\Pr(H\mid\neg E))-(1-\Pr(H\mid E))\Pr(H\mid\neg E)}{\Pr(H\mid E)(1-\Pr(H\mid\neg E))+(1-\Pr(H\mid E))\Pr(H\mid\neg E)}\right)$$

with g:$\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function.

Indeed for any hypothesis H and any evidence E we have:

$$c_{smSB}(H,E)=g\left(\frac{\Pr(H\mid E)(1-\Pr(H\mid\neg E))-(1-\Pr(H\mid E))\Pr(H\mid\neg E)}{\Pr(H\mid E)(1-\Pr(H\mid\neg E))+(1-\Pr(H\mid E))\Pr(H\mid\neg E)}\right)$$

$$=g\left(\frac{\dfrac{\Pr(H\mid E)(1-\Pr(H\mid\neg E))}{(1-\Pr(H\mid E))\Pr(H\mid\neg E)}-1}{\dfrac{\Pr(H\mid E)(1-\Pr(H\mid\neg E))}{(1-\Pr(H\mid E))\Pr(H\mid\neg E)}+1}\right)=g\left(\frac{\left(\dfrac{\dfrac{\Pr(H\mid E)}{\Pr(H)}\times\dfrac{\Pr(\neg H\mid\neg E)}{\Pr(\neg H)}}{\dfrac{\Pr(\neg H\mid E)}{\Pr(\neg H)}\times\dfrac{\Pr(H\mid\neg E)}{\Pr(H)}}\right)-1}{\left(\dfrac{\dfrac{\Pr(H\mid E)}{\Pr(H)}\times\dfrac{\Pr(\neg H\mid\neg E)}{\Pr(\neg H)}}{\dfrac{\Pr(\neg H\mid E)}{\Pr(\neg H)}\times\dfrac{\Pr(H\mid\neg E)}{\Pr(H)}}\right)+1}\right)$$

$$=g\left(\frac{\left(\dfrac{\dfrac{\Pr(E\mid H)}{\Pr(E)}\times\dfrac{\Pr(\neg E\mid\neg H)}{\Pr(\neg E)}}{\dfrac{\Pr(E\mid\neg H)}{\Pr(E)}\times\dfrac{\Pr(\neg E\mid H)}{\Pr(\neg E)}}\right)-1}{\left(\dfrac{\dfrac{\Pr(E\mid H)}{\Pr(E)}\times\dfrac{\Pr(\neg E\mid\neg H)}{\Pr(\neg E)}}{\dfrac{\Pr(E\mid\neg H)}{\Pr(E)}\times\dfrac{\Pr(\neg E\mid H)}{\Pr(\neg E)}}\right)+1}\right)=g\left(\frac{\dfrac{\Pr(E\mid H)\Pr(\neg E\mid\neg H)}{\Pr(E\mid\neg H)\Pr(\neg E\mid H)}-1}{\dfrac{\Pr(E\mid H)\Pr(\neg E\mid\neg H)}{\Pr(E\mid\neg H)\Pr(\neg E\mid H)}+1}\right)$$

$$=g\left(\frac{\Pr(E\mid H)\Pr(\neg E\mid\neg H)-\Pr(E\mid\neg H)\Pr(\neg E\mid H)}{\Pr(E\mid H)\Pr(\neg E\mid\neg H)+\Pr(E\mid\neg H)\Pr(\neg E\mid H)}\right)$$

$$=g\left(\frac{(1-\Pr(\neg E\mid H))\Pr(\neg E\mid\neg H)-(1-\Pr(\neg E\mid\neg H))\Pr(\neg E\mid H)}{(1-\Pr(\neg E\mid H))\Pr(\neg E\mid\neg H)+(1-\Pr(\neg E\mid\neg H))\Pr(\neg E\mid H)}\right)$$

$$=-g\left(\frac{\Pr(\neg E\mid H)(1-\Pr(\neg E\mid\neg H))-(1-\Pr(\neg E\mid H))\Pr(\neg E\mid\neg H)}{\Pr(\neg E\mid H)(1-\Pr(\neg E\mid\neg H))+(1-\Pr(\neg E\mid H))\Pr(\neg E\mid\neg H)}\right)$$

$$=-c_{smSB}(\neg E,H).$$

4) **Hypothesis-inversion symmetry**

Observe that if a confirmation measure $c_{smSB}(H, E)$ satisfies *HIS*, then for any hypothesis H and any evidence E we have:

$$c_{smSB}(H, E) = -c_{smSB}(E, \neg H).$$

A family of confirmation measures $c_{smSB}(H, E)$ that satisfy *HIS* is the following:

$$c_{smSB}(H,E) = g\left(\frac{\Pr(H|E)(1-\Pr(H|\neg E)) - (1-\Pr(H|E))\Pr(H|\neg E)}{\Pr(H|E)(1-\Pr(H|\neg E)) + (1-\Pr(H|E))\Pr(H|\neg E)}\right)$$

with g:$\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function.
Indeed, suppose that $c_{smSB}(H, E)$ satisfies *EIS*. Applying *EIS* three times we get:

$$c_{smSB}(H, E)=-c_{SB}(\neg E, H)=c_{smSB}(\neg H, \neg E)=-c_{smSB}(E, \neg H).$$

Thus, taking into account the first and the last term we conclude that if $c_{smSB}(H, E)$ satisfies *EIS*, then it satisfies also *HIS*. Therefore since the above confirmation measure $c_{smSB}(H, E)$ satisfies *EIS* as proved in the previous point (i.e., point 3)), it satisfies also the hypothesis-inversion symmetry *HIS*.

5) **Inversion symmetry**
Observe that if a confirmation measure $c_{smSB}(H, E)$ satisfies *IS*, then for any hypothesis *H* and any evidence *E* we have:

$$c_{smSB}(H, E) =c_{smSB}(E, H).$$

A family of confirmation measures $c_{smSB}(H, E)$ that satisfy *IS* is the following:

$$c_{smSB}(H,E) = g\left(\frac{\Pr(H|E)(1-\Pr(H|\neg E)) - (1-\Pr(H|E))\Pr(H|\neg E)}{\Pr(H|E)(1-\Pr(H|\neg E)) + (1-\Pr(H|E))\Pr(H|\neg E)}\right)$$

with g:$\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function.
Indeed, by proof in point 3), we get

$$c_{smSB}(H,E) = g\left(\frac{\Pr(H|E)(1-\Pr(H|\neg E)) - (1-\Pr(H|E))\Pr(H|\neg E)}{\Pr(H|E)(1-\Pr(H|\neg E)) + (1-\Pr(H|E))\Pr(H|\neg E)}\right)$$
$$= g\left(\frac{\Pr(E|H)\Pr(\neg E|\neg H) - \Pr(E|\neg H)\Pr(\neg E|H)}{\Pr(E|H)\Pr(\neg E|\neg H) + \Pr(E|\neg H)\Pr(\neg E|H)}\right).$$

(18)

We have also that

$$g\left(\frac{\Pr(E\mid H)\Pr(\neg E\mid\neg H)-\Pr(E\mid\neg H)\Pr(\neg E\mid H)}{\Pr(E\mid H)\Pr(\neg E\mid\neg H)+\Pr(E\mid\neg H)\Pr(\neg E\mid H)}\right)$$
$$=g\left(\frac{\Pr(E\mid H)(1-\Pr(E\mid\neg H))-\Pr(E\mid\neg H)(1-\Pr(E\mid H))}{\Pr(E\mid H)(1-\Pr(E\mid\neg H))+\Pr(E\mid\neg H)(1-\Pr(E\mid H))}\right)=c_{smSB}(E,H)\ . \tag{19}$$

Thus, by (18) and (19) we obtain that for the considered family of confirmation measures:

$$c_{smSB}(H, E) = c_{smSB}(E,H).$$

6) **Evidence-hypothesis symmetry**
Observe that if a confirmation measure $c_{smSB}(H, E)$ satisfies *EHS*, then for any hypothesis H and any evidence E we have:

$$c_{smSB}(H, E) = c_{smSB}(\neg H, \neg E).$$

A family of confirmation measures $c_{smSB}(H, E)$ that satisfy *EHS* is the following:

$$c_{smSB}(H,E)=g(\Pr(H\mid E)-\Pr(H\mid\neg E))$$

with $g:\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function.
Indeed for any hypothesis H and any evidence E we have:

$$\begin{aligned}c_{smSB}(H,E)&=g(\Pr(H\mid E)-\Pr(H\mid\neg E))\\&=g((1-\Pr(H\mid\neg E))-(1-\Pr(H\mid E)))\\&=g(\Pr(\neg H\mid\neg E)-\Pr(\neg H\mid E))=c_{smSB}(\neg H,\neg E).\end{aligned}$$

Observe also that if $c_{smSB}(H, E)$ satisfies *EIS*, then it satisfies also *EHS*, because, indeed, for any hypothesis H and any evidence E, applying *EIS* twice, we have:

$$c_{smSB}(H, E) = -c_{smSB}(\neg E, H)=c_{SB}(\neg H,\neg E).$$

Consequently, a confirmation measure $c_{smSB}(H, E)$ satisfying *EIS* will satisfy also *EHS*, so that *EHS* is satisfied also by the family of confirmation measures considered in the above point 3), that is:

$$c_{smSB}(H,E)=g\left(\frac{\Pr(H\mid E)(1-\Pr(H\mid\neg E))-(1-\Pr(H\mid E))\Pr(H\mid\neg E)}{\Pr(H\mid E)(1-\Pr(H\mid\neg E))+(1-\Pr(H\mid E))\Pr(H\mid\neg E)}\right)$$

with $g:\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function.

7) **Evidence-hypothesis-inversion symmetry**
Observe that if a confirmation measure $c_{smSB}(H, E)$ satisfies *EHIS*, then for any hypothesis H and any evidence E we have:

$$c_{smSB}(H, E) = c_{smSB}(\neg E, \neg H).$$

A family of confirmation measures $c_{smSB}(H, E)$ that satisfy *EHIS* is the following:

$$c_{smSB}(H,E) = g\left(\frac{\Pr(H \mid E)(1-\Pr(H \mid \neg E)) - (1-\Pr(H \mid E))\Pr(H \mid \neg E)}{\Pr(H \mid E)(1-\Pr(H \mid \neg E)) + (1-\Pr(H \mid E))\Pr(H \mid \neg E)}\right) \tag{20}$$

with $g:\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function.
Indeed observe that a confirmation measure $c_{smSB}(H, E)$ satisfying *IS* and *EHS* satisfies also *EHIS*, because by applying first *IS* and then *EHS* we get:

$$c_{smSB}(H, E) = c_{smSB}(E,H) = c_{smSB}(\neg E,\neg H).$$

Thus, we have

$$c_{smSB}(H, E) = c_{smSB}(\neg E,\neg H).$$

Consequently, taking into account what we proved in above points 5) and 6), the family of confirmation measures (20) satisfies both *IS* and *EHS*, and thus, it also satisfies the evidence-hypothesis-inversion symmetry *EHIS*.

As the final part of this proof, let us show now that there are confirmation measures $c_{smSB}(H, E)$ that satisfy all symmetry properties.
Observe that given any confirmation measure $c(H, E)$

- if $c(H, E)$ satisfies *EIS* and *IS*, then $c(H, E)$ satisfies also *ES*,
- if $c(H, E)$ satisfies *HIS* and *IS*, then $c(H, E)$ satisfies also *HS*,
- if $c(H, E)$ satisfies *EHIS* and *IS*, then $c(H, E)$ satisfies also *EHS*.

Indeed,

- if $c(H, E)$ satisfies *EIS* and *IS*, applying firstly *EIS* and after *IS*, we get
$$c(H, E) = -c(\neg E, H) = -c(\neg H,E),$$
so that, taking the first and last term we conclude that $c(H, E)$ satisfies also *ES*;
- if $c(H, E)$ satisfies *HIS* and *IS*, applying firstly *HIS* and after *IS*, we get
$$c(H, E) = -c(E,\neg H) = -c(\neg H, E),$$
so that, taking the first and last term we conclude that $c(H, E)$ satisfies also *HS*;
- if $c(H, E)$ satisfies *EHIS* and *IS*, applying firstly *EHIS* and after *IS*, we get
$$c(H, E) = c(\neg E,\neg H) = c(\neg H, \neg E),$$

so that, taking the first and last term we conclude that $c(H, E)$ satisfies also *EHS*.

Thus observing that a family of confirmation measures:

$$c_{smSB}(H,E) = g\left(\frac{\Pr(H \mid E)(1-\Pr(H \mid \neg E)) - (1-\Pr(H \mid E))\Pr(H \mid \neg E)}{\Pr(H \mid E)(1-\Pr(H \mid \neg E)) + (1-\Pr(H \mid E))\Pr(H \mid \neg E)}\right)$$

with $g:\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function, satisfies *IS*, *EIS*, *HIS* and *EHIS*, we have to conclude that it also satisfies *ES*, *HS* and *EHS*. Thus, it satisfies all the symmetry properties.

It is worthwhile to observe that, in case g is the identity, i.e. $g(x)=x$, this confirmation measure is in fact the Yule Q index, because

$$\frac{\Pr(H \mid E)(1-\Pr(H \mid \neg E)) - (1-\Pr(H \mid E))\Pr(H \mid \neg E)}{\Pr(H \mid E)(1-\Pr(H \mid \neg E)) + (1-\Pr(H \mid E))\Pr(H \mid \neg E)}$$

$$= \frac{\frac{a}{a+c}\frac{d}{b+d} - \frac{c}{a+c}\frac{b}{b+d}}{\frac{a}{a+c}\frac{d}{b+d} + \frac{c}{a+c}\frac{b}{b+d}} = \frac{ad-bc}{ad+bc}.$$

□

A9. Proof of Theorem 8

Theorem 8. A confirmation measure $c(H,E) = f(\Pr_L, \Pr_R)$ being strictly monotonic in the converse Bayesian or the converse likelihoodist perspective is not strictly monotonic in the other perspectives.

Proof. Consider again scenario 1 corresponding to the contingency table in Table 1 and scenario 2 corresponding to the contingency table in Table 2. As already observed in the proof of above Theorem 1, for any confirmation measure $c_{smB}(H,E)$ strictly monotonic in the Bayesian perspective we have

$$c_{smB}(H_1,E_1) = c_{smB}(H_2,E_2).$$

Observe also that $\Pr(H_1)=\Pr(H_2)=0.2$ and that $\Pr(H_1|\neg E_1)=0.114$ while $\Pr(H_2|\neg E_2)=0$, so that for any converse Bayesian confirmation measure $c_{smCB}(H,E)$ strictly monotonic in the converse Bayesian perspective we get

$$c_{smCB}(H_1,E_1) < c_{smCB}(H_2,E_2).$$

As a result, $c_{smCB}(H,E)$ cannot respect the strict monotonicity in the Bayesian perspective. With analogous examples one can prove the thesis for all remaining perspectives. One can proceed similarly with confirmation measures $c_{smCL}(H,E)$ being strictly monotonic in the converse likelihoodist perspective. □

A10. Proof of Lemma 1

Lemma 1. If $c(H, E)$ is a confirmation measure monotonic in the perspective P, then $c^X(H, E)$ is in the perspective showed in Table 5, P∈{Bayesian (*B*), strong Bayesian (*SB*), likelihoodist (*L*), strong likelihoodist (*SL*), converse Bayesian (*CB*), converse likelihoodist (*CL*)}, $X \in \{ES,HS,EIS,HIS,IS,EHS,EHIS\}$.

Table 5. Transformation of perspectives of confirmation measures after negation and/or inversion of evidence *E* and hypothesis *H*

***X*\P**	***B***	***SB***	***L***	***SL***	***CB***	***CL***
ES	*CB*	*SB*	*L*	*SL*	*B*	*CL*
HS	*B*	*SB*	*CL*	*SL*	*CB*	*L*
EIS	*L*	*SL*	*CB*	*SB*	*CL*	*B*
HIS	*CL*	*SL*	*B*	*SB*	*L*	*CB*
IS	*L*	*SL*	*B*	*SB*	*CL*	*CB*
EHS	*CB*	*SB*	*CL*	*SL*	*B*	*L*
EHIS	*CL*	*SL*	*CB*	*SB*	*L*	*B*

Proof. Suppose $c(H, E)$ is a confirmation measure monotonic in the Bayesian perspective. Then there exist f:[0,1]×[0,1]→$\mathfrak{R}$ non-decreasing with the first argument and non-increasing with the second argument, with $f(x,x) = 0$ for all $x\in[0,1]$, such that $c(H,E) = f(\Pr(H|E), \Pr(H))$. Thus we have:

- $c^{ES}(H,E) = -c(H,\neg E) = -f(\Pr(H|\neg E), \Pr(H))$, which is non-increasing in $\Pr(H|\neg E)$ and non-decreasing in $\Pr(H)$, so that $c^{ES}(H,E)$ is in the converse Bayesian perspective;
- $c^{HS}(H,E) = -c(\neg H, E) = - f(1-\Pr(H|E), 1-\Pr(H))$, which is non-decreasing in $\Pr(H|E)$ and non-increasing in $\Pr(H)$, so that $c^{HS}(H,E)$ is in the Bayesian perspective;
- $c^{EIS}(H,E) = -c(\neg E, H) = -f(1-\Pr(E|H), 1-\Pr(E))$, which is non-decreasing in $\Pr(E|H)$ and non-increasing in $\Pr(E)$, so that it $c^{EIS}(H,E)$ in the likelihoodist perspective;
- $c^{HIS}(H,E) = -c(E,\neg H) = -f(\Pr(E|\neg H), \Pr(E))$, which is non-increasing in $\Pr(E|\neg H)$ and non-decreasing in $\Pr(E)$, so that $c^{HIS}(H,E)$ is in the converse likelihoodist perspective;
- $c^{IS}(H,E) = c(E,H) = f(\Pr(E|H), \Pr(E))$, which is non-decreasing in $\Pr(E|H)$ and non-increasing in $\Pr(E)$, so that $c^{IS}(H,E)$ is in the likelihoodist perspective;
- $c^{EHS}(H,E) = c(\neg H,\neg E) = f(1-\Pr(H|\neg E), 1-\Pr(H))$, which is non-increasing in $\Pr(H|\neg E)$ and non-decreasing in $\Pr(H)$, so that $c^{EHS}(H,E)$ is in the converse Bayesian perspective;
- $c^{EHIS}(H,E) = c(\neg E ,\neg H) = f(1-\Pr(E|\neg H), 1-\Pr(E))$, which is non-increasing in $\Pr(E|\neg H)$ and non-decreasing in $\Pr(E)$, so that $c^{EHIS}(H,E)$ is in the converse likelihoodist perspective.

The other cases can be proved analogously. □

A11. Proof of Lemma 2

Lemma 2. For any transformation $c^X(H, E)$ of $c(H, E)$, where $X\in\{ES,HS,EIS,HIS,IS,EHS,EHIS\}$, there exists an inverse transformation X^{-1} such that $c^{XX^{-1}}(H,E) = c^{X^{-1}X}(H,E) = c(H,E)$.

More precisely we have:

- $ES^{-1}=ES$,
- $HS^{-1}=HS$,
- $EIS^{-1}=HIS$,
- $HIS^{-1}=EIS$,
- $IS^{-1}=IS$,

- EHS^{-1}=EHS,
- $EHIS^{-1}$=$EHIS$.

Proof. Let us observe, that applying the *ES* twice we have

$$c^{ES\,ES}(H, E) = -c^{ES}(H, \neg E) = c(H, E),$$

so that we can conclude that ES^{-1}=ES.
The other cases can be proved analogously.
Observe that for $X\in\{ES,HS,IS,EHS,EHIS\}$, we get X=X^{-1}, so that we get

$$c^{XX^{-1}}(H,E)=c^{XX}(H,E)=c^{X^{-1}X}(H,E)=c(H,E).$$

Moreover, we have

$$c^{EIS\,HIS}(H,E)=c^{HIS\,EIS}(H,E)=c(H,E)$$

so that, also for $X\in\{EIS,HIS\}$, we get

$$c^{XX^{-1}}(H,E)=c^{X^{-1}X}(H,E)=c(H,E). \ \square$$

A12. Proof of Lemma 3

Lemma 3. Given confirmation measures $c(H, E)$ and $c_1(H, E)$ and $X,Y\in\{ES,HS,EIS,HIS,IS,EHS,EHIS\}$, such that $c(H, E)=c_1^X(H,E)$, confirmation measure $c_1(H, E)$ satisfies symmetry property Y, that is $c_1(H, E)=c_1^Y(H,E)$, if and only if confirmation measure $c(H, E)$ satisfies property XYX^{-1}, that is $c(H, E)=c^{XYX^{-1}}(H,E)$.

Proof. Suppose that $c_1(H, E)$ satisfies symmetry property Y and therefore $c_1(H, E)=c_1^Y(H,E)$. Remembering that $c(H, E)=c_1^X(H,E)$, we get

$$c(H, E)=c_1^X(H,E)= c_1^{XY}(H,E)=c^{XYX^{-1}}(H,E).$$

Thus, we proved that if $c_1(H, E)$ satisfies symmetry property Y, then $c(H, E)$ satisfies symmetry property XYX^{-1}.
Suppose now that $c(H, E)$ satisfies symmetry property XYX^{-1}. Observe that by $c(H, E)=c_1^X(H,E)$, we get $c^{X^{-1}}(H,E)=c_1(H,E)$.
Thus we have

$$c_1(H,E)=c^{X^{-1}}(H,E)=c^{X^{-1}XYX^{-1}}(H,E)=$$

$$c^{YX^{-1}}(H,E)=c_1^{YX^{-1}X}(H,E)=c_1^Y(H,E).$$

Thus, we proved that if $c(H, E)$ satisfies symmetry property XYX^{-1}, then $c_1(H, E)$ satisfies symmetry property Y. □

A13. Proof of Theorem 9

Theorem 9. Consider a confirmation measure

- $c_{smL}(H, E)$ strictly monotonic in the likelihoodist perspective , i.e., being strictly increasing with respect to $\Pr(E|H)$ and strictly decreasing with respect to $\Pr(E)$,
- $c_{smCB}(H, E)$ strictly monotonic in the converse Bayesian perspective, i.e., being strictly decreasing with respect to $\Pr(H|\neg E)$ and strictly increasing with respect to $\Pr(H)$,
- $c_{smCL}(H, E)$ strictly monotonic in the converse likelihoodist perspective, i.e., being strictly decreasing with respect to $\Pr(E|\neg H)$ and strictly increasing with respect to $\Pr(E)$.

We have that:

1) there exist confirmation measures $c_{smL}(H, E)$ and $c_{smCL}(H, E)$ that satisfy *ES*, while there are no confirmation measures $c_{smCB}(H, E)$ that satisfy *ES*,
2) there exist confirmation measures $c_{smCB}(H, E)$ that satisfy *HS*, while there are no confirmation measures $c_{smL}(H, E)$ and $c_{smCL}(H, E)$ that satisfy *HS*;
3) there are no confirmation measures $c_{smL}(H, E)$, $c_{smCB}(H, E)$ and $c_{smCL}(H, E)$ that satisfy *EIS*;
4) there are no confirmation measures $c_{smL}(H, E)$, $c_{smCB}(H, E)$ and $c_{smCL}(H, E)$ that satisfy *HIS*;
5) there exist confirmation measures $c_{smL}(H, E)$, $c_{smCB}(H, E)$ and $c_{smCL}(H, E)$ that satisfy *IS*;
6) there are no confirmation measures $c_{smL}(H, E)$, $c_{smCB}(H, E)$ and $c_{smCL}(H, E)$ that satisfy *EHS*;
7) there exist confirmation measures $c_{smL}(H, E)$, $c_{smCB}(H, E)$ and $c_{smCL}(H, E)$ that satisfy *EHIS*.

Moreover, if $c_{smCB}(H, E)$ ($c_{smL}(H, E)$ or $c_{smCL}(H, E)$) satisfies one among *HS*, *IS* and *EHIS* (one among *ES*, *IS* and *EHIS*), it cannot satisfy any of the remaining two symmetry properties. Finally, there are confirmation measures $c_{smL}(H, E)$, $c_{smCB}(H, E)$ and $c_{smCL}(H, E)$ that do not satisfy any symmetry property.

Proof. Observe that if $c_{smB}(H, E)$ is a confirmation measure strictly monotonic in the Bayesian perspective, then, by Lemma 1 $c_{smL}(H, E) = c_{smB}{}^{IS}(H,E)$ is a confirmation measure strictly monotonic in the likelihoodist perspective. Thus, by Lemma 3, remembering that by Lemma 2 $IS^{-1} = IS$, we have that $c_{smL}(H, E)$ satisfies a symmetry property *IS Y IS* if and only if $c_{smB}(H, E)$ satisfies the symmetry property *Y*. By Theorem 6

$c_{smB}(H, E)$ can satisfy one among *HS*, *IS* and *EHIS*. Consequently, $c_{smL}(H, E)$ can satisfy one of the following three symmetry properties

- *IS HS IS* = *ES*,
- *IS IS IS* = *IS*,
- *IS EHIS IS* = *EHIS*.

We can prove that *IS HS IS* = *ES* as follows. For any confirmation measure $c(H, E)$ we get
$c^{IS\,HS\,IS}(H, E) = c^{HS\,IS}(E, H) = -c^{IS}(\neg E, H) = -c(H, \neg E) = c^{ES}(H, E)$.
The other two cases, can be proved analogously.

To prove that any confirmation measure $c_{smL}(H, E)$ strictly monotonic in the likelihoodist perspective cannot satisfy properties *HS*, *EIS*, *HIS* and *EHS* we can use two contingency tables obtained from those in Table 3 and Table 4 by applying to the variables transformation *IS* (i.e. *E* exchanges with *H* and $\neg E$ exchanges with $\neg H$), which permits to pass from a confirmation measure $c_{smB}(H, E)$ strictly monotonic in the Bayesian perspective to a confirmation measure $c_{smL}(H, E)$ strictly monotonic in the likelihoodist perspective. We can prove that any confirmation measure $c_{smL}(H, E)$ cannot satisfy *HS*, *EIS*, *HIS* and *EHS*, by using the so obtained contingency tables and adopting arguments analogous to those used in Theorem 6 to prove that any confirmation measure $c_{smB}(H, E)$ does not satisfy *ES*, *EIS*, *HIS* and *EHS*.

Proceeding in the same way, we can prove the properties satisfied and not satisfied by confirmation measures $c_{smCB}(H, E)$ and $c_{smCL}(H, E)$. In particular,

- for confirmation measures $c_{smCB}(H, E)$, remembering that by Lemma 1 there exists a confirmation measure $c_{smB}(H, E)$ such that $c_{smCB}(H,E) = -c_{smB}^{ES}(H,E)$ and that by Lemma 2 $ES^{-1}=ES$, by Lemma 3 we get that $c_{smCB}(H, E)$ can satisfy one of the following three symmetry properties:
  - *ES HS ES* = *HS*,
  - *ES IS ES* = *EHIS*,
  - *ES EHIS ES* = *IS*;
- for confirmation measures $c_{smCL}(H, E)$, remembering that by Lemma 1 there exists a confirmation measure $c_{smB}(H, E)$ such that $c_{smCL}(H, E) = -c_{smB}^{HIS}(H,E)$ and that by Lemma 2 $HIS^{-1}=EIS$, by Lemma 3 we get that $c_{smCL}(H, E)$ can satisfy one of the following three symmetry properties:
  - *HIS HS EIS* = *ES*,
  - *HIS IS EIS* = *EHIS*,
  - *HIS EHIS EIS* = *IS*.

To prove that any confirmation measure $c_{smCB}(H, E)$ cannot satisfy properties *ES*, *EIS*, *HIS* and *EHS* we can use two contingency tables obtained from those ones in Table 3 and Table 4 by applying to the variables transformation *ES* (i.e. E exchanges with $\neg E$), which permits to pass from a confirmation measure $c_{smB}(H, E)$ to a confirmation measure $c_{smCB}(H, E)$.

Analogously, to prove that any confirmation measure $c_{smCL}(H, E)$ cannot satisfy properties *HS*, *EIS*, *HIS* and *EHS* we can use two contingency tables obtained from those in Table 3 and Table 4 by applying to the variables transformation *HIS* (i.e. H exchanges with E and E exchanges with $\neg H$), which permits to pass from a confirmation measure $c_{smB}(H, E)$ to a confirmation measure $c_{smCL}(H, E)$.

On the basis of this last result, proceeding in the same way as in Theorem 6 where we proved the analogous statement for confirmation measures $c_{smB}(H, E)$ strictly monotonic in the Bayesian perspective, we can prove that if $c_{smCB}(H, E)$ ($c_{smL}(H, E)$ or $c_{smCL}(H, E)$) satisfies one among *HS*, *IS* and *EHIS* (one among *ES*, *IS* and *EHIS*), it cannot satisfy any of the remaining two symmetry properties. For example, to prove that a confirmation measure $c_{smL}(H, E)$ satisfying symmetry property *ES* cannot satisfy also symmetry property *IS*, we can proceed as follows. By contradiction suppose that $c_{smL}(H, E)$ satisfies properties *ES* and *IS*. Applying first *ES* and after *IS*, we get

$$c_{smL}(H, E) = -c_{smL}(H, \neg E) = -c_{smL}(\neg E, H).$$

Thus, taking into account the first and the last term, we have that $c_{smL}(H, E)$ satisfies *EIS*. But we have already proved that this is impossible and, therefore, it is also not possible that $c_{smL}(H, E)$ satisfies at the same time both *ES* and *IS* properties.

Finally, here are confirmation measures $c_{smL}(H, E)$, $c_{smCB}(H, E)$ and $c_{smCL}(H, E)$ that do not satisfy any symmetry property:

$$c_{smL}(H, E) = \sqrt{\Pr(E \mid H)} - \sqrt{\Pr(E)},$$

$$c_{smCB}(H, E) = \sqrt{\Pr(H)} - \sqrt{\Pr(H \mid \neg E)},$$

$$c_{smL}(H, E) = \sqrt{\Pr(E)} - \sqrt{\Pr(E \mid \neg H)}.$$

Proceeding analogously as in the final part of the proof of Theorem 6, one can show that they do not satisfy any of the considered symmetry properties.

□

A14. Proof of Theorem 10

Theorem 10. Consider a confirmation measure $c_{smSL}(H, E)$ strictly monotonic in the strong likelihoodist perspective, i.e., being strictly increasing with respect to Pr(E|H) and strictly decreasing with respect to Pr(¬E| H). For any symmetry property there are confirmation measures $c_{smSL}(H, E)$ satisfying it. Moreover, there are confirmation measures $c_{smSL}(H, E)$ that satisfy all symmetry properties.

Proof. Observe that if $c_{smSL}(H, E)$ is a confirmation measure strictly monotonic in the likelihoodist perspective, then, by Lemma 1 $c_{smSL}^{IS}(H, E)$ is a confirmation measure strictly monotonic in the strong Bayesian perspective. Thus, by Lemma 3, we have that $c_{smSL}(H, E)$ satisfies a symmetry property if and only if $c_{smSL}^{IS}(H, E)$ satisfies the same symmetry property. Consequently, the confirmation measure $c_{smSL}(H, E)$ satisfies or does not satisfy the same symmetry properties as confirmation measure strictly monotonic in the strong Bayesian perspectives as presented in Theorem 7.

For the same reasons, a family of confirmation measures:

$$c_{smSL}(H,E) = g\left(\frac{\Pr(E\,|\,H)(1-\Pr(E\,|\,\neg H)) - (1-\Pr(E\,|\,H))\Pr(E\,|\,\neg H)}{(1-\Pr(E\,|\,H))\Pr(E\,|\,\neg H) + (1-\Pr(E\,|\,H))\Pr(E\,|\,\neg H)}\right)$$

with g:$\mathfrak{R}\rightarrow\mathfrak{R}$ being a strictly increasing odd function, satisfies all the symmetry properties. □